airloads and the case of the finite span airloads arises only when the unsteady lifts and moments are used to calculate the generalized (modal) forces. At this point note, of course, that the basic strip theory analyses were the same as the finite span analyses, but with the finite span correction matrices omitted.

There was a second variation in style from the procedure for calculating the finite span airloads outlined on pp. 14 and 15 of Ref. 2. The tables of Ref. 2 are set up for calculating the finite span unsteady airloads at every two-tenths of the semispan. In the case of the model wings, there would only be five points of definition for the airloads, and the airloads would be based on only a five point description of the natural modes. Since p. 1 of Ref. 2 states "...the effect of finite span depends appreciably on the shape of the wing deflection functions," and since the modal amplitudes would have been poorly described by just a five point definition, the author decided to abandon the tables of Ref. 2 and retain the nine point definitions of the mode shapes. Thus the unsteady airloads were calculated with better accuracy at their natural locations, i.e., the midpoints of the panel quarterchords. The price that had to be paid for this decision was the necessity for calculating the Reissner functions $S_n(k_os,\phi_i)$ defined in Ref. 2 where n is a series index number, k_os is a reduced frequency referenced to the semispan length, and $\cos\phi_i$ is a nondimensional spanwise coordinate. The complex values of the functions S_n are difficult to calculate because those values depend upon an integral involving Cicala's function (see Ref. 2, p. 37). While the imaginary portion of Cicala's function presents no difficulty on a finite interval, the real part is unbounded as the argument approaches zero. This produces a discontinuity in the integrand of the imaginary part of S_n, such that at values of the variable of integration slightly less than the value at the point of discontinuity, the integrand logarithmically approaches plus infinity, while at values of the variable of integration slightly greater than that at the point of discontinuity, the integrand logarithmically approaches minus infinity. This integrand was integrated on a digital computer by using straight line interpolation along with the values of Cicala's function tabulated in Ref. 2 down to an argument value of 0.05. For lesser values of the Cicala function argument, it was assumed that the sharp rise in opposite directions produced a zero net value for the integral across the discontinuity. This questionable assumption was investigated by reproducing three of the four S_n tables that appear in Ref. 2. Generally there was good agreement, but there were also points of disagreement. The disagreements were mostly among the more difficult imaginary parts, but it is significant that they also occurred among the real parts. For example, the real part of S_7 (2, arccos 0.2) from Ref. 2 is -0.299. A hand calculation using a polynomial approximation presented in Ref. 2 yielded the value -0.1269. A desk top sized graphical solution produced -0.1273, and the digital computer program gave -0.1250. Here the computer program, as opposed to the tables of Ref. 2, is clearly favored. A case that was not quite as clear was the imaginary portion of S_5 (2, arccos 0.8). The table of Ref. 2 says $+0.115$; a desk top sized graph said $+0.0175$; and the computer said $+0.0051$. The final justification for the use of the digital computer values of S_n can only be the reasonable calculated flutter airspeeds that were obtained. See Table 1.

The flutter airspeed results indicate a marked improvement in analytical accuracy when the Reissner theory results are contrasted to the basic strip theory results. In two of the ten cases, however, the finite span solutions were slightly nonconservative. The basic strip theory results evidenced their usual wide range of conservativeness. Thus it, would appear that having both types of results would be useful. All that can be said for the flutter frequency results is that the Reissner theory values are less than the wind-tunnel values. Graphs of this small number of data points indicate the that Reissner solutions follow the experimental trends for variations in aspect ratio, taper ratio and thickness ratio. At the high sweepback angle of 55°, however, the trends were opposite.

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Drag of a Flat Plate with Transition in the Absence of Pressure Gradient

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Introduction

BASED on a more sound analysis than Schlichting's[1] formula, Collar[2] obtained a closed formula for the drag of a flat plate with transition in the absence of pressure gradient But Collar's formula gives no better agreement with existing data than the Schlichting formula. The present Note shows that, if the logarithmic skin-friction relation is used instead of the skin-friction relation deduced from the ⅐th power law velocity distribution, a better agreement can be obtained.

Analysis

Similar to the analysis given by Collar, Fig. 1 shows the model. X_c is the transition point and X_i is the notional leading edge of a plate which in wholly turbulent flow gives an identical turbulent layer from X_c to L. Assuming that the turbulent layer from X_i to X_c follows the ⅐th power law and the momentum thickness is continuous at X_c, we have

$$0.664(\nu X_c/U)^{1/2} = 0.036(X_c - X_i)\{[U(X_c - X_i)]/\nu\}^{1/5} \quad (1)$$

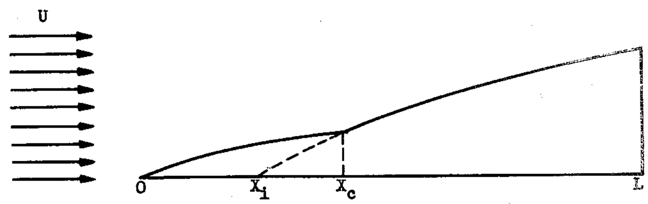

Fig. 1 The model.

Received April 10, 1972.
Index category: Airplane and Component Aerodynamics.

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After some arrangements, Eq. (1) becomes

$$R_{ci} = (18.44\, R_c^{1/2})^{5/4} \tag{2}$$

Where

$$R_{ci} = [U(X_c - X_i)]/\nu$$
$$R_c = UX_c/\nu$$

The contribution of drag due to laminar part from $X = 0$ to X_c is

$$D_1 = 0.644b(X_c \mu \rho U^3)^{1/2} \tag{3}$$

Where b is the width of the plate. Whereas the contribution of drag due to turbulent part is

$$D_t = \frac{0.455}{(\log R_t)^{2.58}} \tfrac{1}{2}\rho b U^2 X_t - \frac{0.455}{(\log R_{ci})^{2.58}} \tfrac{1}{2}\rho b U^2 (X_c - X_i) \tag{4}$$

Where

$$X_t = L - (X_c - X_i) \quad \text{or} \quad R_t = R_L - R_{ci} \qquad R_t = UX_t/\nu$$

Then, the total skin-friction coefficient is defined as

$$C_f = (D_1 + D_t)/\tfrac{1}{2}\rho b L U^2 \tag{5}$$

With Eqs. (3) amd (4) substituted into Eq. (5), we have

$$C_f = 1.328\left(\frac{R_c}{R_L}\right)^{1/2} + \frac{0.455}{R_L}\left[\frac{R_t}{(\log R_t)^{2.58}} - \frac{R_{ci}}{(\log R_{ci})^{2.58}}\right] \tag{6}$$

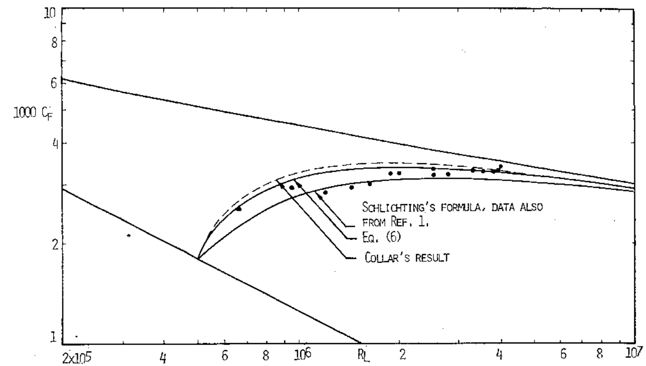

Fig. 2 Values of C_f

where $R_L = UL/\nu$. Given a value of R_c, R_{ci} can be calculated by Eq. (2). Then C_f can be evaluated by Eq. (6). Figure 2 shows a typical result for $R_c = 5 \times 10^5$. Schlichting's[1] as well as Collar's[2] results are also shown. It is evident that Eq.(6) fit the experimental data better than other methods.

References

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